Cumhuriyet Sci. J., 44(1) (2023) 160-169
DOI: https://doi.org/10.17776/csj.1120077

Cumhuriyet Science Journal

| csj.cumhuriyet.edu.tr | Founded: 2002 ISSN: 2587-2680 e-ISSN: 2587-246X Publisher: Sivas Cumhuriyet University

On Solutions Of Random Partial Differential Equations With Laplace Adomian Decomposition Method

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Research Article

History
Received: 23/05/2022
Accepted: 05/02/2023

ABSTRACT

In this study, random partial differential equations obtained by randomly choosing the coefficients or initial conditions of partial differential equations will be analyzed. With the help of Laplace Adomian Decomposition Method and Homotopy Analysis Method, approximate analytical solutions of random partial differential equations were obtained. Initial conditions and parameters are made into random variables with normal distribution and gamma distribution. Probability characteristics such as expected value, variance and confidence intervals of the obtained random partial differential equation are calculated. Obtained results will be plotted with the help of MATLAB (2013a), package program and random results will be interpreted.

Keywords: Random partial differential equation, Normal distribution, Laplace-Adomian decomposition method, Gamma distribution.

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Introduction

The Adomian Decomposition Method was first introduced by George Adomian in the early 1980s. Adomian is an American mathematician who developed this method for ordinary, partial, linear and nonlinear differential equations. Adomian applied this method to find approximate solutions to deterministic, stochastic, linear and nonlinear problems with boundary and initial conditions. The method is constructed by decomposing its nonlinear terms. It is defined as $Ny = \sum_{n=0}^{\infty} A_n$. Here, A_n are Adomian polynomials. Each A_n depends on the arguments $y_0, y_1, y_2, \dots, y_n$ for $n > 0$. The formulas to obtain these polynomials were developed by Adomian. Finding approximate analytical solutions of non-linear differential equations with the Adomian Decomposition Method will make the solution faster and more reliable in many areas mentioned above.

In this study, Laplace Adomian Decomposition Method (LADM) is used to calculate approximate solutions of nonlinear ordinary and partial differential equations. Laplace-Adomian Decomposition Method (LADM) is a combination of Adomian Decomposition Method and Laplace Transform Method. This method has been successfully used to solve different problems in [1-7]. Also, LADM does not require predefined dimension definition like the Runge–Kutta method. Also, LADM can be considered as an ideal method for ordinary and partial equations representing nonlinear models. Compared to other analytical methods, LADM has fewer parameters, so LADM is a useful technique that does not require discretization and linearization [8]. A comparison between LADM and ADM for analysis of FDEs is given in [9]. The Kundu-Eckhaus equation deals with quantum field theory and the analytical solution of these nonlinear PDEs is explored in [10] using LADM. The multi-step Laplace Adomian decomposition method for nonlinear FDEs is described in [11]. The analysis of the smoke model was successfully studied using LADM [12].

The motivation of this study is the previous literature on random modeling of several diseases (Merdan et al., 2017; Merdan et al., 2018; Bekiryazici and Hasimoglu et al., 2022). Gamma and Normal (Gauss) distributions will be used for the distributions of the random parameters.

The aim of this study is to analyze the solution behavior graphically by finding various probability characteristics such as expected value, variance and confidence intervals by obtaining the approximate analytical solution of random partial differential equations with the use of the Laplace-Adomian Method.

Adomian Decomposition Method Combined with Laplace Transform

Let the partial or ordinary differential equation $Fy(x,t) = g(x,t)$ be given with the initial condition $y(x,0) = f(x)$. Here F is the differential operator with linear and non-linear terms. In this case, the operator form is defined by

$$L_t y(x,t) + Ry(x,t) + Ny(x,t) = g(x,t) \tag{1}$$

$L_t = \frac{\partial}{\partial t}$, R is a linear operator with partial derivatives with respect to x, N is a nonlinear operator, and g is an inhomogeneous term independent of y.

The solution for $L_t y(x,t)$ can be expressed as

$$L_t y(x,t) = g(x,t) - Ry(x,t) - Ny(x,t) \quad (2)$$

The $\mathcal{L}$ Laplace transform is an integral transform found by Pierre-Simon Laplace. It is a powerful and practical method for solving ordinary and partial differential equations.

Definitions and Theorems

Definition 1. Given the $f(t)$ function for every $t \geq 0$; Let f be defined [13] as the Laplace transform F. Therefore,

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt$$

The Laplace transform of the t^n function is found as follows:

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

Definition 2. Given a continuous function $f(t)$, if $F(s) = L\{f(t)\}$ then $f(t)$ is called the inverse Laplace transform of $F(s)$ and

$$f(t) = \mathcal{L}^{-1}\{F(s)\} \quad (3)$$

It is expressed by (3). The Laplace transform has derivative properties:

$$\mathcal{L}\{f^{(n)}(t)\} = s^n\mathcal{L}\{f(t)\} - \sum_{k=0}^{n-1} s^{n-1-k} f^{(k)}(0)$$

$$\mathcal{L}\{t^n f(t)\} = (-1)^n F^{(n)}(s)$$

The Laplace Adomian Decomposition Method consists of applying the Laplace transform to both sides of the equation (2).

$$\mathcal{L}\{L_t y(x,t)\} = \mathcal{L}\{g(x,t)\} - \mathcal{L}\{Ry(x,t)\} - \mathcal{L}\{Ny(x,t)\}$$

$$sy(x,s) - y(x,0) = \mathcal{L}\{g(x,t) - Ry(x,t) - Ny(x,t)\}$$

$$y(x,s) = \frac{f(x)}{s} - \frac{1}{s}\mathcal{L}\{-g(x,t) + Ry(x,t) + Ny(x,t)\}$$

Equation (2) is found and inverse Laplace transform is applied to this equation,

$$y(x,t) = f(x) - \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\{-g(x,t) + Ry(x,t) + Ny(x,t)\}\right] \quad (4)$$

is obtained.
The Adomian Decomposition Method produces a series of solutions given by $y(x,t)$:

$$y(x,t) = \sum_{n=0}^{\infty} y_n(x,t) \quad (5)$$

$$Ny(x,t) = \sum_{n=0}^{\infty} A_n(y_0, y_1, \dots, y_n) \quad (6)$$

is a sequence of Adomian polynomials. A_n Adomian polynomials,

$$A_0 = f(y_0)$$

$$A_1 = y_1\frac{df(y_0)}{dy_0}$$

$$A_2 = y_2\frac{df(y_0)}{dy_0} + \frac{y_1^2}{2!}\frac{d^2f(y_0)}{dy_0^2}$$

$$A_3 = y_3\frac{df(y_0)}{dy_0} + y_1y_2\frac{d^2f(y_0)}{dy_0^2} + \frac{y_1^3}{3!}\frac{d^3f(y_0)}{dy_0^3}$$

$$A_4 = y_4 \frac{df(y_0)}{dy_0} + \left(\frac{1}{2!}y_2^2 + y_1 y_3\right)\frac{d^2 f(y_0)}{dy_0^2} + \frac{1}{2!}y_1^2 y_2 \frac{d^3 f(y_0)}{dy_0^3} + \frac{y_1^4}{4!}\frac{d^4 f(y_0)}{dy_0^4}$$

...

form can be obtained. With $\lambda \in R$ being the parameter,

$$y = \sum_{n=0}^{\infty} y_n$$

solution series of the function,

$$y = \sum_{n=0}^{\infty} \lambda^n A_n$$

and nonlinear

$$f(y) = \sum_{n=0}^{\infty} \lambda^n A_n$$

can be written parametrically. Adomian Polynomials can be obtained from the formula (7), provided that the $f(y)$ function at the $\lambda \in R$ point is analytical.

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} f\left(\sum_{i=0}^{\infty} \lambda^i y_i\right)\right]_{\lambda=0}, n \geq 0 \tag{7}$$

Using the equations (4), (5) and (6),

$$\sum_{n=0}^{\infty} y_n(x,t) = f(x) - \mathcal{L}^{-1}\left[-\frac{1}{s}\mathcal{L}\{g(x,t)\} + \frac{1}{s}\mathcal{L}\{R\sum_{n=0}^{\infty} y_n(x,t) + \sum_{n=0}^{\infty} A_n(y_0, y_1, \ldots, y_n)\}\right] \tag{8}$$

is obtained. From equation (8), the following formulas are subtracted:

$$\begin{cases} y_0(x,t) = f(x) \\ y_{n+1}(x,t) = -\mathcal{L}^{-1}\left[-\frac{1}{s}\mathcal{L}\{g(x,t)\} + \frac{1}{s}\mathcal{L}\{Ry_n(x,t) + A_n(y_0, y_1, \ldots, y_n)\}\right], n = 0,1,2 \ldots \end{cases} \tag{9}$$

Using equation (9), an approximate solution is obtained:

$$y(x,t) \approx \sum_{n=0}^{k} y_n(x,t)$$

$$\lim_{k\to\infty} \sum_{n=0}^{k} y_n(x,t) = y(x,t)$$

Application

To examine the solution behavior of random partial differential equations, an approximate solution is obtained by the Laplace-Adomian Decomposition Method. Various interpretations were made by establishing the expected value, variance and %99 confidence interval of the solution.

Example 1.

Consider the following random partial differential equation

$$y_t + y_{xx} - y^2 - y \cdot y_{xx} = 0 \tag{10}$$

subject to the initial conditions

$$y(x,0) = Bsinx \tag{11}$$

where $B \sim N(\mu, \sigma^2)$ is parameter with Normal distribution.

To solve (10)-(11) by means of Laplace-Adomian Decomposition Method, making the Laplace transform of Equation (10).

When the operations in (1)-(4) are performed, the following relation is obtained.

$$\begin{cases} y_0(x,t) = f(x) \\ y_{n+1}(x,t) = -\mathcal{L}^{-1}\left[-\frac{1}{s}\mathcal{L}\{g(x,t)\} + \frac{1}{s}\mathcal{L}\{Ry_n(x,t) + A_n(y_0, y_{1,} \ldots, y_n)\}\right], n = 0,1,2, \ldots \end{cases} \tag{12}$$

Identifying the zeroth component $y_0(x,t)$ by $Bsinx$, the remaining components $y_n(x,t)$, $n \geq 1$, can be determined by using the recurrence relation

$$y_0(x,t) = Bsinx$$

$$A_0 = f(y_0) = {y_0}^2 = B^2 sin^2 x$$

where A_n are Adomian polynomials that represent the nonlinear term and other terms are as follows

$$A_1 = 2y_0 y_1 = 2B^2 t sin^2 x$$
$$A_2 = 2y_0 y_2 + {y_1}^2 = B^2 t^2 sin^2 x + \frac{B^2 t^2}{3} sin^2 x(3 - 2B(t-3)sinx) \tag{13}$$
…

Other polynomials can be generated similarly to enhance the accuracy of the approximation. Using equations (12) and (13) above, the following connections are found.

$$y_1(x,t) = \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\{Ry_0(x,t) + A_0\}\right] = Btsinx$$

$$y_2(x,t) = \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\{Ry_1(x,t) + A_1\}\right] = \frac{Bt^2}{6} sinx(3 - 2B(t-3)sinx)$$
$$y_3(x,t) = \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\{Ry_2(x,t) + A_2\}\right] = -\frac{2B^4t^5}{315} sin^4x(5t^2 - 35t + 63) + \frac{Bt^3}{1260}\Big[210sinx + 21B^2 sin^3x(5t^3 - 18t^2 - 10t + 40) + B\Big(2cos^2x\Big(-420 + 105t - Bt^2 sinx\big(35t - 126 - 4Bsinx(5t^2 - 35t + 63)\big)\Big) - 21sin^2x(3t^2 + 10t - 80)\Big)\Big]$$

…

If the $y_0(x,t)$, $y_1(x,t)$, $y_2(x,t)$, $y_3(x,t)$ found above are written in the series, the following relation is obtained.

$$y_{LADM} = y_0(x,t) + y_1(x,t) + y_2(x,t) + y_3(x,t) + \cdots \tag{14}$$

If the values $y_0(x,t)$, $y_1(x,t)$, $y_2(x,t)$, $y_3(x,t)$ are written and edited in (14),

$$y_{LADM} = Bsinx + Btsinx + \frac{Bt^2}{6} sinx(3 - 2B(t-3)sinx) - \frac{2B^4t^5}{315} sin^4x(5t^2 - 35t + 63) + \frac{Bt^3}{1260}\Big[210sinx + 21B^2 sin^3x(5t^3 - 18t^2 - 10t + 40) + B\Big(2cos^2x\Big(-420 + 105t - Bt^2 sinx\big(35t - 126 - 4Bsinx(5t^2 - 35t + 63)\big)\Big) - 21sin^2x(3t^2 + 10t - 80)\Big)\Big] + \cdots \tag{15}$$

is obtained. The solution in a series form is given by y_{LADM} and using Taylor series, the exact solution

$$y(x,t) = Bsinx(1 + \frac{t}{1!} + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots) \tag{16}$$

is readily obtained.

$N[\phi(x,t;q)]$ nonlinear operator, $L[\phi(x,t;q)]$ L linear operator and $L\big(c_1(x)\big) = 0$, $c_1(x)$ integration constant; apply Homotopy Analysis Method [15-17] to equation (10) given with $y_0(x,t) = Bsinx$ initial condition.

$$N[\phi(x,t;q)] = \frac{\partial\phi(x,t;q)}{\partial t} + \frac{\partial^2\phi(x,t;q)}{\partial x^2} - \phi(x,t;q)^2 - \phi(x,t;q)\frac{\partial^2\phi(x,t;q)}{\partial x^2}$$
$$L[\phi(x,t;q)] = \frac{\partial\phi(x,t;q)}{\partial t}$$
$$L\big(c_1(x)\big) = 0,\ c_1(x)$$

Equation (18) is obtained by using equation (17).

$$L[y_m(t) - \chi_m y_{m-1}(t)] = hH(t)R_m(\vec{y}_m, t) \tag{17}$$
$$R_m[\vec{y}_{m-1}] = \frac{\partial y_{m-1}(x,t)}{\partial t} + \frac{\partial^2 y_{m-1}(x,t)}{\partial x^2} - y_{m-1}(x,t)^2 - y_{m-1}(x,t)\frac{\partial^2 y_{m-1}(x,t)}{\partial x^2} \tag{18}$$

The solution of equation (17) for $m \geq 1$ deformation of order m.

$$y_m(x,t) = \chi_m y_{m-1}(x,t) + hH(r,t)L^{-1}[R_m(\vec{y}_{m-1}(x,t))]$$

If $m \geq 1$, $\chi_m = 1$, $h = -1$ ve $H(r,t) = 1$,

$$R_1(\vec{y}_0(x,t)) = \frac{\partial y_0(x,t)}{\partial t} + \frac{\partial^2 y_0(x,t)}{\partial x^2} - y_0(x,t)^2 - y_0(x,t)\frac{\partial^2 y_0(x,t)}{\partial x^2} = -B\sin(x) \tag{19}$$

$$y_1(x,t) = \chi_1 y_0(x,t) + \hbar L^{-1}[R_1 \vec{y}_0(x,t)]$$
$$y_1(x,t) = -Bsin(x) + \hbar \int (-Bsin(x))dt$$
$$y_1(x,t) = B\sin x + Bt\text{sin}x$$
$$y_2(x,t) = \chi_1 y_1(x,t) + \hbar L^{-1}[R_2 \vec{y}_1(x,t)]$$
$$y_2(x,t) = \frac{1}{6}\text{Bsin}\, x(6 + 6t + 3t^2 + 2B\sin x)$$
$$y_3(x,t) = \chi_1 y_2(x,t) + \hbar L^{-1}[R_3 \vec{y}_2(x,t)]$$
$$y_3(x,t) = -\frac{1}{18}B(-3t^3\sin x - 18\sin x - 18t\sin x + B^2t^3\sin x + 3B^2t^2\sin x + 6B^2t\sin x - 9B^2t^2\sin x\, cosx^2 - 18B^2t\sin x\, cosx^2 - 3B^2t^3\text{sinxcosx}^2 - 9t^2\text{sinx} - 6B + 2B^3t + 6B^3tcosx^4 + 6Bcosx^2 - 8B^3tcosx^2 + 24Btcosx^2 - 12Bt)$$

Continuing in this way, the closed form solution is found as (20).

$$y(x,t) = Bsinx(1 + \frac{t}{1!} + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots) \tag{20}$$

Again, it has been observed that the solution obtained by Laplace-Adomian Decomposition Method is compatible with each other when compared to the Homotopy Analysis Method solution.

The parameters of the normally distributed random variable X are $B{\sim}N(\mu,\sigma^2)$. Using the moment-generating function of the normal distribution, we get

$$M_X(t) = E[e^{tX}] = e^{\frac{1}{2}\sigma^2t^2+\mu t} \tag{21}$$

from (17), the 1st and 2nd moment of the random variable $B{\sim}N(\mu,\sigma^2)$ are,

$$E[B] = \mu, \quad E[B^2] = \sigma^2 + \mu^2,$$

is calculated as. If the basic properties of the expected value for the X and Y independent random variables are used, the expected value of equations (16) and (20) is:

$$E[y(x,t)] = \left(x + tx + \tfrac{1}{2}t^2x - \tfrac{1}{6}x^3 + \cdots\right)E(B) = \left(x + tx + \tfrac{1}{2}t^2x - \tfrac{1}{6}x^3 + \cdots\right) \tag{22}$$

The expected value (22) is obtained. If $B{\sim}N(\mu = 2, \sigma^2 = 4)$ is specially selected,

$$E[y(x,t)] = 2\left(x + tx + \tfrac{1}{2}t^2x - \tfrac{1}{6}x^3 + \cdots\right)$$

is obtained. If the expected value is plotted with MATLAB (2013a) for the given parameter values, the graph in Figure 1. is obtained.

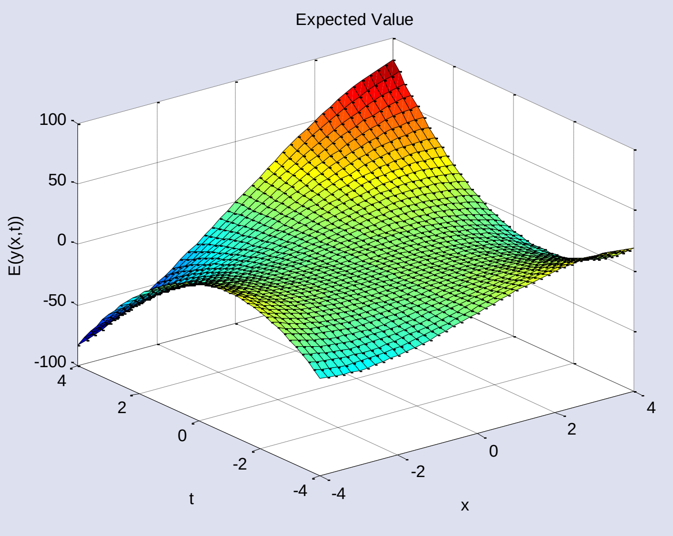

Figure 1. The solution behavior of the expected value of equations (16) and (20) for $B{\sim}N(\mu = 2, \sigma^2 = 4)$ special values..

If the basic properties of the variance are used for each random variable X, the variance of equations (16) and (20) is calculated as (24).

$$Var(B) = E(B^2) - [E(B)]^2 = \sigma^2 + \mu^2 - \mu^2 = \sigma^2 \tag{23}$$
$$Var[y(x,t)] = \left(x + tx + \frac{1}{2}t^2x - \frac{1}{6}x^3 + \cdots\right)^2 Var(B)$$

If the value in (23) is substituted,

$$Var[y(x,t)] = \left(x + tx + \frac{1}{2}t^2x - \frac{1}{6}x^3 + \cdots\right)^2 \sigma^2$$

is obtained. If $B \sim N(\mu = 2, \sigma^2 = 4)$ is specially selected,

$$Var[y(x,t)] = 4\left(x + tx + \frac{1}{2}t^2x - \frac{1}{6}x^3 + \cdots\right)^2$$

is obtained. Figure 2. is obtained if the variance is plotted with MATLAB (2013a) for the given parameter values.

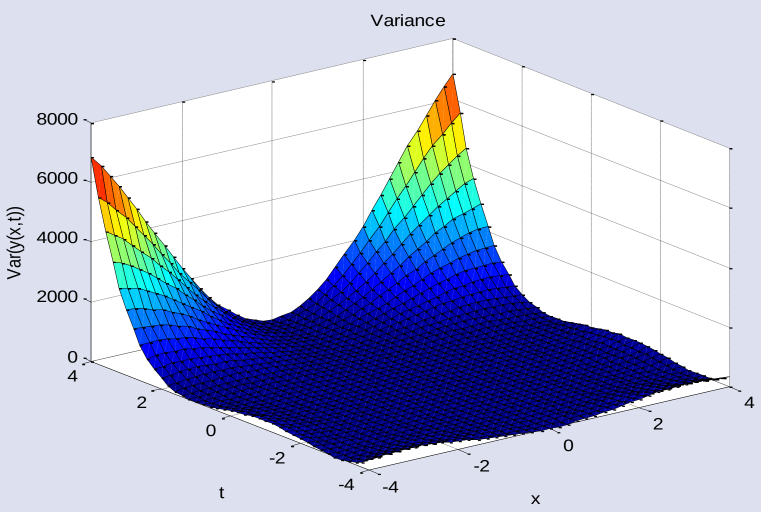

Figure 2. Solution behavior of the variance of equations (16) and (20) for $B \sim N(\mu = 2, \sigma^2 = 4)$ special values..

The expected value and variance values for $x = 0.5$ are given in Table 1.

Table 1. Table for the expectation value and variance with $x = 0.5$

t	$E[y(x,t)]$	$Var[y(x,t)]$
0.0	0.9583333333	0.9184027780
0.1	1.063333333	1.130677778
0.2	1.178333333	1.388469445
0.3	1.303333333	1.698677778
0.4	1.438333333	2.068802778
0.5	1.583333333	2.506944445
0.6	1.738333333	3.021802778
0.7	1.903333333	3.622677778
0.8	2.078333333	4.319469448
0.9	2.263333333	5.122677780
1.0	2.458333333	6.043402780

The standard deviation is equal to the square root of the variance.

$$std\left(y(t)\right) = \sqrt{Var\left(y(t)\right)} \tag{24}$$

Confidence intervals for expected values of random variables,

$$(E(y(t)) - K.std\left(y(t)\right), E\left(y(t)\right) + K.std(y(t)) \tag{25}$$

is equation to and this can be obtained through standard deviations. For $K = 3$, this formula gives approximately %99 confidence interval for the approximate expected value of the normally distributed random variable [14]. If the %99 confidence interval is plotted with MATLAB (2013a), the graph in Figure 3. is obtained.

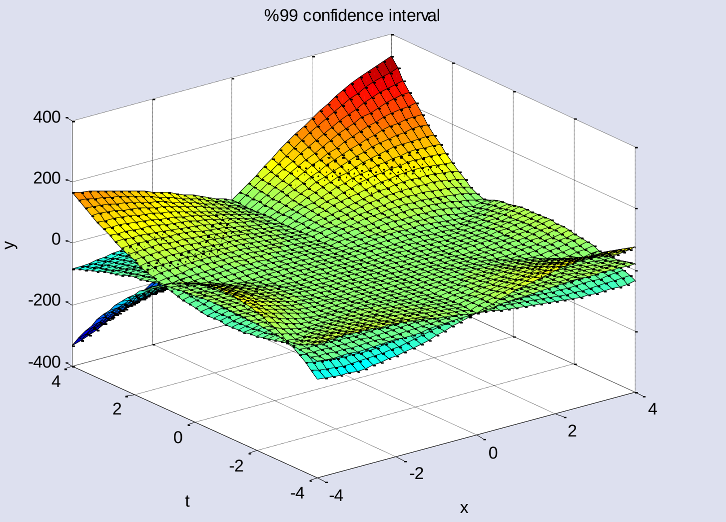

Figure 3. The solution behavior of the %99 confidence interval of equations (16) and (20) for $B \sim N(\mu = 2, \sigma^2 = 4)$ special values..

Example 2.

We will then consider the following random partial differential equation

$$By_t = By_{xx} + 6By - 6y^2 \tag{26}$$

random partial differential equation is subject to the following initial conditions.

$$y(x,0) = \frac{B}{(1+e^x)^2} \tag{27}$$

Where $B \sim Gamma(\omega, \lambda)$ is parameter with Gamma distribution.

To solve (26)-(27) by means of Laplace-Adomian Decomposition Method, making the Laplace transform of equation (26).

By applying the present method,

$$\begin{aligned}
y_0(x,t) &= \frac{B}{(1+e^x)^2} \\
y_1(x,t) &= \mathcal{L}^{-1}\left[-\frac{1}{s}\mathcal{L}\left\{y_{0,xx} + 6y_0 - \frac{6}{B}{y_0}^2\right\}\right] = -\frac{10Bt(e^{2x}+e^x)}{(1+e^x)^4} \\
y_2(x,t) &= \mathcal{L}^{-1}\left[-\frac{1}{s}\mathcal{L}\left\{y_{1,xx} + 6y_1 - \frac{A_1}{B}\right\}\right] = \frac{25Bt^2(2e^{2x}-e^x)}{(1+e^x)^4} \\
y_3(x,t) &= \mathcal{L}^{-1}\left[-\frac{1}{s}\mathcal{L}\left\{y_{2,xx} + 6y_2 - \frac{A_2}{B}\right\}\right] = -\frac{125Bt^3(4e^{3x}-7e^{2x}+e^x)}{3(1+e^x)^5}
\end{aligned} \tag{28}$$

$$\begin{aligned}
A_0 &= 6{y_0}^2 = \frac{6B^2}{(1+e^x)^4} \\
A_1 &= 12y_0y_1 = -120\frac{B^2t(e^{2x}+e^x)}{(1+e^x)^6}
\end{aligned} \tag{29}$$

$$A_2 = 12y_0y_2 + 6{y_1}^2 = \frac{600B^2t^2(e^{2x}+e^x)}{(1+e^x)^8} + \frac{300B^2t^2(2e^{2x}-e^x)}{(1+e^x)^6}$$

...

equations are obtained.

If the values $y_0(x,t),\ y_1(x,t),\ y_2(x,t),\ y_3(x,t)$ are written and edited in (14),

$$y_{LADM} = \frac{B}{(1+e^x)^2} - \frac{10Bt(e^{2x}+e^x)}{(1+e^x)^4} + \frac{25Bt^2(2e^{2x}-e^x)}{(1+e^x)^4} - \frac{125Bt^3(4e^{3x}-7e^{2x}+e^x)}{3(1+e^x)^5} + \cdots \tag{30}$$

equation (30) is obtained. If the equation opens to Taylor series;

$$y(x,t) = \frac{1}{4}B - \frac{1}{4}Bx + \frac{5}{4}Bt + \frac{1}{16}Bx^2 - \frac{5}{8}Btx + \frac{25}{16}Bt^2 + \frac{1}{48}Bx^3 - \frac{5}{16}Btx^2 + \frac{25}{16}Bt^2x - \frac{125}{48}Bt^3 + \cdots \tag{31}$$

solution is obtained. If the random variable X has a gamma distribution, using the moment generating function,

$$M_X(t) = E[e^{tX}] = \left(\frac{\lambda}{\lambda - t}\right)^{\alpha}$$

from the expression, the 1st and 2nd moment of the random variable $B \sim Gamma(\omega,\lambda)$,

$$E[B] = \frac{\omega}{\lambda},\ E[B^2] = \frac{\omega(\omega+1)}{\lambda^2}$$

is calculated as. If the basic properties of the expected value for the X and Y independent random variables are used, the expected value of equation (31) is

$$E[y(x,t)] = \left(\frac{1}{4} - \frac{1}{4}x + \frac{5}{4}t + \frac{1}{16}x^2 - \frac{5}{8}tx + \frac{25}{16}t^2 + \frac{1}{48}x^3 - \frac{5}{16}tx^2 + \frac{25}{16}t^2x - \frac{125}{48}t^3 + \cdots\right)E(B) \tag{32}$$

is obtained. If the moment value found above is substituted in the expression (32),

$$E[y(x,t)] = \left(\frac{1}{4} - \frac{1}{4}x + \frac{5}{4}t + \frac{1}{16}x^2 - \frac{5}{8}tx + \frac{25}{16}t^2 + \frac{1}{48}x^3 - \frac{5}{16}tx^2 + \frac{25}{16}t^2x - \frac{125}{48}t^3 + \cdots\right)\frac{\omega}{\lambda} \tag{33}$$

the expected value is found as (33). If $B \sim Gamma(\omega = 2, \lambda = 3)$ is specially selected,

$$E[y(x,t)] = \left(\frac{1}{4} - \frac{1}{4}x + \frac{5}{4}t + \frac{1}{16}x^2 - \frac{5}{8}tx + \frac{25}{16}t^2 + \frac{1}{48}x^3 - \frac{5}{16}tx^2 + \frac{25}{16}t^2x - \frac{125}{48}t^3 + \cdots\right)\frac{2}{3}$$

is obtained. If the expected value is plotted with MATLAB (2013a) for the given parameter values, the graph in Figure 4. is obtained.

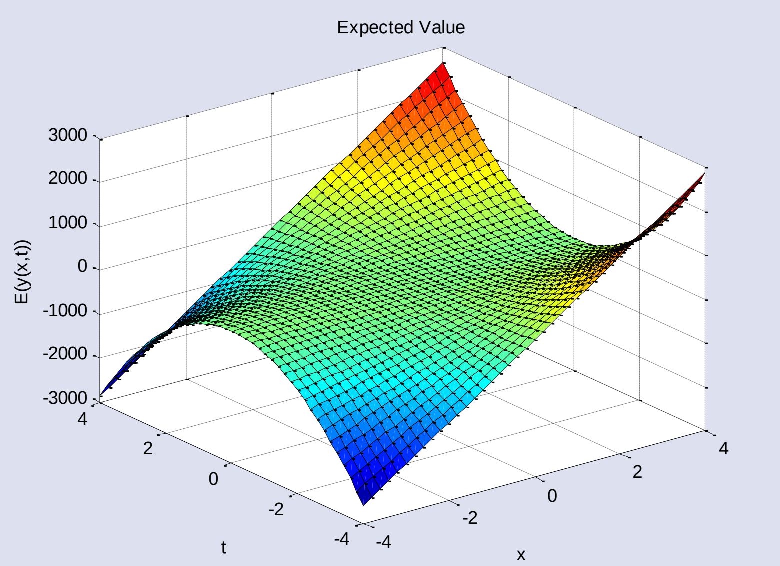

Figure 4. The solution behavior of the expected value of equation (31) for $B \sim Gamma(\omega = 2, \lambda = 3)$ special values..

$$Var(B) = \frac{\omega(\omega+1)}{\lambda^2} - \left(\frac{\omega}{\lambda}\right)^2 = \frac{\omega}{\lambda^2} \quad (34)$$

$$Var[y(x,t)] = \left(\frac{1}{4} - \frac{1}{4}x + \frac{5}{4}t + \frac{1}{16}x^2 - \frac{5}{8}tx + \frac{25}{16}t^2 + \frac{1}{48}x^3 - \frac{5}{16}tx^2 + \frac{25}{16}t^2x - \frac{125}{48}t^3 + \cdots\right)^2 Var(B) \quad (35)$$

To calculate the variance of equation (31), if (34) is substituted in (35), the variance value is calculated as follows.

$$Var[y(x,t)] = \left(\frac{1}{4} - \frac{1}{4}x + \frac{5}{4}t + \frac{1}{16}x^2 - \frac{5}{8}tx + \frac{25}{16}t^2 + \frac{1}{48}x^3 - \frac{5}{16}tx^2 + \frac{25}{16}t^2x - \frac{125}{48}t^3 + \cdots\right)^2 \frac{\omega}{\lambda^2}$$

If $B \sim Gamma(\omega = 2, \lambda = 3)$ is specially selected,

$$Var[y(x,t)] = \left(\frac{1}{4} - \frac{1}{4}x + \frac{5}{4}t + \frac{1}{16}x^2 - \frac{5}{8}tx + \frac{25}{16}t^2 + \frac{1}{48}x^3 - \frac{5}{16}tx^2 + \frac{25}{16}t^2x - \frac{125}{48}t^3 + \cdots\right)^2 \frac{2}{9}$$

is obtained. Figure 5. is obtained if the variance is plotted with MATLAB (2013a) for the given parameter values.

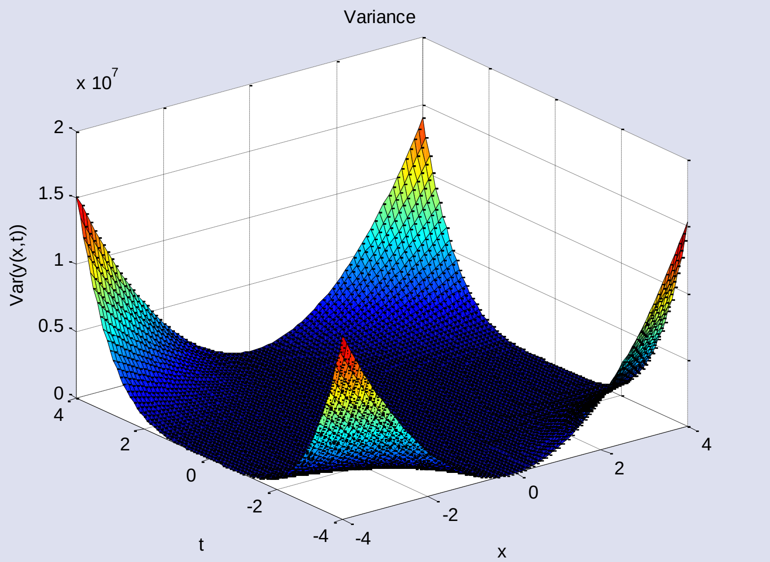

Figure 5. The solution behavior of the variance of equation (31) for $B \sim Gamma(\omega = 2, \lambda = 3)$ special values..

The expected value and variance values for $x = 0.5$ are given in Table 2.

Table 2. Table for the expectation value and variance with $x = 0.5$

$\mathbf{t}$	$\boldsymbol{E[y(x,t)]}$	$\boldsymbol{Var[y(x,t)]}$
0.0	0.09548611114	0.004558798709
0.1	0.1666666667	0.01388888889
0.2	0.2586805555	0.03345781493
0.3	0.3611111111	0.06520061729
0.4	0.4635416667	0.1074354384
0.5	0.5555555556	0.1543209878
0.6	0.6267361110	0.1963990766
0.7	0.6666666669	0.2222222222
0.8	0.6649305551	0.2210663222
0.9	0.6111111110	0.1867283952
1.0	0.4947916670	0.1224093967

If (24) and (25) equations are used, for $K = 3$; If the interval $(0, 2.05)$ is taken into account, the %98 confidence interval of the expected value of the random variable showing the Gamma distribution is plotted with MATLAB (2013a), and the graph in Figure 6. is obtained.

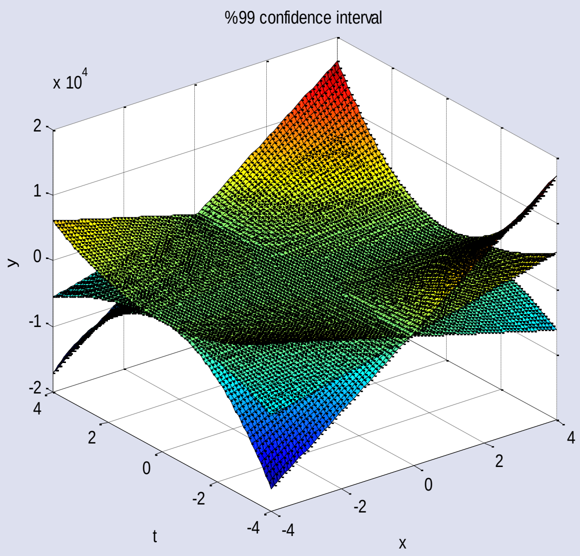

Figure 6. The solution behavior of the %99 confidence interval of equation (31) for $B \sim Gamma(\omega = 2, \lambda = 3)$ special values..

Conclusions

In this study, a hybrid method Laplace Addomain Decomposition Method is applied to find the solution of some important partial differential equations which are randomized with the help of Normal and Gamma distributions. We conclude that the Laplace Addomain Decomposition Method (LADM) is a powerful and efficient technique that can be used to find the approximate analytical solution of nonlinear random partial differential equations. The analytical solution of the given problem is given with the help of power series. The initial conditions or coefficients of Random PDD were selected from the Normal and Gamma distribution, and expected value, variance and confidence intervals, which are the main probability characteristics, were obtained to analyze random effects.

Acknowledgments

The authors are grateful to referees for careful reading, suggestions, and valuable comments, which have substantially improved the paper.

Conflicts of interest

There are no conflicts of interest in this work.

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