

**Research Article****Proposed new equations for calculation of thermophysical properties of nanofluids****Mahmut Kaplan**<sup>a,\*</sup> and **Melda Özdiñç Çarpınlioğlu**<sup>b</sup> <sup>a</sup>Amasya University, Technology Faculty, Department of Mechanical Engineering, Amasya, 05100, Turkey<sup>b</sup>Gaziantep University, Faculty of Engineering, Department of Mechanical Engineering, Gaziantep, 27310, Turkey

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## ABSTRACT

A trial-error procedure is applied for the derivation of correlations to estimate the relative thermal conductivity ( $k_r$ ) and dynamic viscosity ( $\mu_r$ ) of nanofluids using MATLAB. Thermophysical properties of particles and base fluids, particle diameter ( $d_p$ ), sphericity, capping layer thickness, Brownian motion of a particle, temperature, and volume fraction ( $\phi$ ) are considered. The accuracy of predicting  $k_r$  and  $\mu_r$  of nanofluids is developed using dimensionless parameters involving base fluid and particle characteristics. The results reveal that the estimated values are in a good agreement with the experimental data with a standard deviation of 2.16% and 8.16% for  $k_r$  and  $\mu_r$  of nanofluids, respectively. Besides that, 97.5% of the predicted  $k_r$  values suit experimental data of  $k_r$  with a mean deviation of  $\pm 5\%$ , whereas 90.4% of the estimated  $\mu_r$  values match the data of  $\mu_r$  with a mean deviation of  $\pm 10\%$ . Therefore, the proposed new equations will be useful for numerical simulation studies and the engineering design of heat transfer devices such as refrigeration systems, solar collectors, and heat exchangers.

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**1. Introduction**

Nanofluids consist of addition of nanometer sized (0-100 nm) particles such as oxides, nitrides, carbides, metal, carbon-based materials and hybrid nanoparticles to base fluids containing water (W), ethylene glycol (EG), ethylene glycol-water (EG-W), oil, etc. Recent review papers on thermophysical properties of nanofluids illustrated that thermal conductivity and dynamic viscosity of nanofluids depend on various parameters including the Brownian velocity, particle size and shape, type of nanoparticle and base fluid, nanofluid temperature ( $T$ ), particle volume fraction ( $\phi$ ), and surfactants [1-5].

The relative thermal conductivity ( $k_r$ ) equals thermal conductivity of nanofluids over thermal conductivity of base fluids. Several researchers [6-18] have improved various correlations for predicting  $k_r$  of nanofluids. The Maxwell model [6] was improved to predict  $k_r$  of dilute suspensions of non-interacting spheres. Hamilton et al. [7] developed a model involving a shape factor by changing the Maxwell model. But both Maxwell's and Hamilton's

models do not consider the influence of temperature and particle size on  $k_r$  of nanofluids. Since the models considering only the influence of variation of  $\phi$  fail to estimate  $k_r$  of nanofluids, many new models including different properties of nanofluids have been improved in recent decades. Koo et al. [8] and Vajjha et al. [10] used a two-term model considering the conventional static conductivity (Maxwell model) and the thermal conductivity thanks to Brownian effect including particle diameter ( $d_p$ ),  $\phi$ ,  $T$ , and properties of base fluid. Patel et al. [11] suggested an empirical equation including change of thermal conductivity of base fluids and particles,  $T$  and  $d_p$  using the non-linear regression analysis. Hassani et al. [14] suggested a correlation including the size distribution of nanoparticles, the Brownian motion,  $\phi$  and  $T$  using the Vaschy-Buckingham theorem.

The dynamic viscosity of nanofluids is also important for the heat transfer applications thanks to the internal resistance to fluid flow and a negative impact on the pumping power. The relative dynamic viscosity of

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nanofluids ( $\mu_r$ ) equals dynamic viscosity of nanofluids divided by dynamic viscosity of base fluids.

Various models [19-27] were suggested to predict  $\mu_r$  of nanofluids. Einstein [19] was the first researcher who developed a theoretical model to estimate  $\mu_r$  of the solid-liquid suspension for  $\phi < 0.2\%$ . Graham [20] presented a model to calculate  $\mu_r$  depending on  $d_p$  and the minimum distance separating the sphere surfaces for low  $\phi$ . Masoumi et al. [22] described a new correlation involving the Brownian motion of nanoparticles,  $d_p$ , particle density, dynamic viscosity of base fluid, and distance between centers of particles. Hosseini et al. [23] improved a model with capping layer thickness, hydrodynamic volume fraction and  $d_p$  on the nanoparticle and  $T$ . Corcione [12] proposed an equation accounting for  $d_p$ , equivalent molecular diameter of the base fluid and  $\phi$ . Adio et al. [24] suggested a correlation considering  $d_p$ ,  $\phi$ , capping layer thickness and  $T$  by using dimensional analysis. The goal of the present paper is to represent new empirical correlating equations for predicting  $k_r$  and  $\mu_r$  of W, EG and EG-W based nanofluids including spherical and cylindrical nanoparticles. Most of researchers have developed their models to estimate  $k_r$  of nanofluids with spherical particles using only their own data. On other hand, the models for estimating  $\mu_r$  of nanofluids are relatively few compared to  $k_r$  models and most of them deal with only change of  $\phi$ . But  $k_r$  and  $\mu_r$  of nanofluids are dependent on many factors. Therefore, the suggested correlations will help to improve predicting  $k_r$  and  $\mu_r$  of nanofluids with extending range by using different dimensionless parameters consisting of the Brownian velocity, thermophysical properties of base fluids and nanoparticles,  $d_p$ ,  $\psi$ ,  $T$ , and  $\phi$ . A summary of different models on  $k_r$  and  $\mu_r$  of nanofluids is presented in Table 1 and Table 2, respectively.

## 2. Methodology

### 2.1 Correlations for Relative Thermal Conductivity

The correlation for predicting  $k_r$  of nanofluids is derived from the experimental data [28, 29, 31-39] including spherical and cylindrical particles with W, EG and EG-W based nanofluids by trial error method using MATLAB.

A second-degree polynomial is used to describe  $k_r$  of nanofluids:

$$k_r = (\beta_2\phi^2 + \beta_1\phi)\Delta_{k_r} + \beta_0 \quad (1)$$

where the terms,  $\beta_2$ ,  $\beta_1$ , and  $\beta_0$  are the empirical constants, and  $\Delta_{k_r}$  is the dimensionless term, which is multiplied by the constants,  $\beta_2$ , and  $\beta_1$  to express the impacts of base fluids and particles characteristics on  $k_r$  of nanofluids.

The term,  $\Delta_{k_r}$  in Equation (1) can be written as:

$$\Delta_{k_r} = \left(\frac{k_p}{k_{bf}}\right)^{0.78} \left[ \ln\left(\frac{k_p}{k_{bf}}\right)^{1.25} \right] \left(\frac{h}{d_p}\right)^{1.12} \left(\frac{T}{T_{ref}}\right)^{13.9} \left(\frac{c_{p_{bf}}}{c_{p_{ref}}}\right)^{5.35} \left(\frac{1}{\psi}\right)^{0.017(d_p/h)} \text{Re}^{0.1} \text{Pr}^{0.24} \quad (2)$$

where  $h$  is capping layer thickness and  $h = 1$  nm [23],  $T_{ref}$  is a reference temperature and  $T_{ref} = 293$  K,  $c_{p_{bf}}$  and  $c_{p_{ref}}$  are specific heat of base fluid and reference specific heat and  $c_{p_{ref}}$  is equal to 4182.2 J/kg (specific heat of water at 293 K) [40].

The particle sphericity,  $\psi$  in Equation (2) is equal to the sphere surface area including equivalent volume as the actual particle [41].  $\psi = 0.5$  and  $\psi = 1$  for cylindrical and spherical nanoparticles, respectively.

The nanoparticle Reynolds number is calculated by:

$$\text{Re} = \frac{\rho_b u_{Br} d_p}{\mu_{bf}} \quad (3)$$

where  $u_{Br}$  is the Brownian velocity, which is computed using the Koo and Kleinstreuer model accounting for the influence of the particle Brownian motion and interaction between particles and fluid molecules [8]:

$$u_{Br} = \sqrt{\frac{18k_b T}{\pi \rho_{bf} d_p^3}} \quad (4)$$

where  $k_b$  is the Boltzmann constant which is equal to  $1.3806 \times 10^{-23}$ .

The Prandtl number of the base fluids is calculated as

$$\text{Pr} = \frac{c_{p_{bf}} \mu_b}{k_{bf}} \quad (5)$$

The constants,  $\beta_2$ ,  $\beta_1$ , and  $\beta_0$  in Equation (1) and the exponents in Equation (2) are determined based on the experimental data [28, 29, 31-39] using MATLAB. For spherical particle with W based nanofluids and cylindrical particle with W, EG and EG-W based fluids,  $\beta_2 = 0.00019$ ,  $\beta_1 = 0.0045$ , and  $\beta_0 = 1.033$ . For spherical particle with EG and EG-W based fluids  $\beta_2 = -0.0008$ ,  $\beta_1 = 0.0716$ , and  $\beta_0 = 1.011$ .

### 2.2 Correlations for Relative Dynamic Viscosity

The correlation for predicting  $\mu_r$  of nanofluids are derived from experimental data [24, 28, 35, 39, 43-48] including spherical and cylindrical particles with W, EG and EG-W based nanofluids by trial error method using MATLAB.

A second-degree polynomial is used to express  $\mu_r$  of nanofluids:

$$\mu_r = (\eta_2\phi^2 + \eta_1\phi)\Delta_{\mu_r} + \eta_0 \quad (6)$$

where the terms,  $\eta_2$ ,  $\eta_1$ , and  $\eta_0$  are the empirical constants, and  $\Delta_{\mu_r}$  is the dimensionless term, which is multiplied by the

constants,  $\eta_2$ , and  $\eta_1$  to express the impacts of base fluids and particles characteristics on  $k_r$  of nanofluids.

The term,  $\Delta\mu_r$  in Equation (6) can be written as:

$$\Delta\mu_r = \left(\frac{\rho_p}{\rho_{bf}}\right)^{2.4} \left(\frac{h}{d_p}\right)^{1.8} \left(\frac{T_{ref}}{T}\right)^8 \left(\frac{1}{\psi}\right)^6 Re^{-0.11} \quad (7)$$

where  $\rho_p$  and  $\rho_{bf}$  are particle and base fluid density. The constants,  $\eta_2$ ,  $\eta_1$ , and  $\eta_0$  in Equation (6) and the exponents in Equation (7) are determined based on the experimental data [24, 28, 35, 39, 43-48] using MATLAB. For spherical

particle with W based nanofluids and cylindrical particle with W, EG and EG-W based fluids,  $\eta_2=0.3136$ ,  $\eta_1=0.5165$ , and  $\eta_0=1.064$ . For spherical particle with EG and EG-W based fluids,  $\eta_2=0.1167$ ,  $\eta_1=0.4717$ , and  $\eta_0=1.064$ .

The thermophysical properties of the base fluids in Equations (2) and (7) are evaluated using the data taken from [40, 42] at the nanofluid temperature. The thermophysical properties of the nanoparticles used for all the calculations are given in Table 3.

Table 1. Various correlations on  $k_r$  nanofluids

Authors	Correlations	Remarks
Maxwell [6]	$k_r = \frac{2k_{bf} + 2(k_p - k_{bf})\phi + k_p}{2k_{bf} - (k_p - k_{bf})\phi + k_p}$	Nanofluids with spherical particles for low $\phi$ .
Hamilton et al. [7]	$k_r = \frac{(n-1)k_{bf} + (n-1)(k_p - k_{bf})\phi + k_p}{(n-1)k_{bf} - (k_p - k_{bf})\phi + k_p}$	$n$ is a shape factor and $n = 3/\psi$ .
Koo et al. [8]	$k_r = \frac{2k_{bf} + 2(k_p - k_{bf})\phi + k_p}{2k_{bf} - (k_p - k_{bf})\phi + k_p} + \frac{5 \times 10^4 \beta \rho_{bf} c_{p,bf} \phi}{k_{bf}} \sqrt{\frac{k_b T}{\rho_p d_p}} f(T, \phi)$ $\beta = 0.0137(100\phi)^{-0.8229} \quad (\phi < 1\%), \quad \beta = 0.0011(100\phi)^{-0.7272} \quad (\phi > 1\%),$ $f(T, \phi) = (0.4705 - 6.04\phi)T + 1722.3\phi - 134.63 \quad \text{for CuO-W mixture.}$	$f(T, \phi)$ is obtained the measured data of Das et al. [9].
Vajjha et al. [10]	$k_r = \frac{2k_{bf} + 2(k_p - k_{bf})\phi + k_p}{2k_{bf} - (k_p - k_{bf})\phi + k_p} + \frac{5 \times 10^4 \beta \rho_{bf} c_{p,bf} \phi}{k_{bf}} \sqrt{\frac{k_b T}{\rho_p d_p}} f(T, \phi)$ $\beta = 8.4407(100\phi)^{-1.07304} \quad (\phi = 1-10\%) \quad \text{for Al}_2\text{O}_3 \text{ and } (\phi = 1-7\%) \quad \text{for ZnO}$ $\beta = 9.881(100\phi)^{-0.9446} \quad (\phi = 1-6\%) \quad \text{for CuO with 60-40\% EG-W}$ $f(T, \phi) = (2.8217 \times 10^{-2} \phi + 3.917 \times 10^{-3}) \frac{T}{T_0} - 3.069 \times 10^{-2} \phi - 3.91123 \times 10^{-3}$	The ranges of $d_p$ , $T$ and $\phi$ are 29-77 nm, 298-363 K and 1-10%.
Patel et al. [11]	$k_r = 1 + 0.135 \left(\frac{k_p}{k_{bf}}\right)^{0.273} \phi^{0.467} \left(\frac{T}{20}\right)^{0.547} \left(\frac{100}{d_p}\right)^{0.234}$	The ranges of $d_p$ , $T$ and $\phi$ are 10-150 nm, 293-343 K and 0.1-3% for nanoparticles with W, EG and transformer oil.
Corcione [12]	$k_r = 4.4 Re^{0.4} Pr^{0.66} \left(\frac{T}{T_{fr}}\right)^{10} \left(\frac{k_p}{k_{bf}}\right)^{0.03} \phi^{0.66} + 1, \quad Re = \frac{\rho_{bf} u_{Br} d_p}{\mu_{bf}}, \quad Pr = \frac{c_{p,bf} \mu_{bf}}{k_{bf}}$	The ranges of $d_p$ , $T$ and $\phi$ are 10-150 nm, 294-324 K and 0.2-9% for Al <sub>2</sub> O <sub>3</sub> , CuO-W and EG, TiO <sub>2</sub> -W and Cu-EG.
Azmi et al. [13]	$k_r = 0.8938 \left(1 + \frac{\phi}{100}\right)^{1.37} \left(1 + \frac{T}{70}\right)^{0.2777} \left(1 + \frac{d_p}{150}\right)^{-0.0336} \left(\frac{\alpha_p}{\alpha_w}\right)^{0.01737}$	$T$ is 394.5 K and the range of $\phi$ is 0-4% nm for Al <sub>2</sub> O <sub>3</sub> (50 nm), ZnO (100 nm) with W.
Hassani et al. [14]	$k_r = 1.04 + \phi^{1.11} \pi_3^{0.33} \pi_4^{-1.7} \left[ \pi_4^{1.7} - 262 \pi_3^{-0.33} + (135 \pi_5^{0.23} \pi_6^{0.82} \pi_7^{-0.1} \pi_8^{-7}) \right]$ $\pi_3 = \frac{k_p}{k_{bf}}, \quad \pi_4 = \frac{c_{p,bf} \mu_{bf}}{k_{bf}}, \quad \pi_5 = \frac{d_{ref}}{d_p}, \quad \pi_6 = \frac{v_{bf}}{d_p u_{Br}}, \quad \pi_7 = \frac{c_{p,bf}}{T^{-1} u_{Br}^2}, \quad \pi_8 = \frac{T_b}{T}$	The ranges of $d_p$ , $T$ and $\phi$ are 15-200 nm, 293-393 K and 0.005-14% for various nanofluids.
Garoosi [17]	$k_r = \frac{2k_{bf} + 2(k_p - k_{bf})\phi + k_p}{2k_{bf} - (k_p - k_{bf})\phi + k_p} + 3.762 \left(\frac{T}{T_0}\right)^{8.661} \left(\frac{d_p}{d_{bf}}\right)^{-0.4351} \left(\frac{k_p}{k_{bf}}\right)^{0.08235} \phi^{0.64} e^{(-5.742\phi)}$ $\omega = (1 + 0.8946 \times \phi)$	Empirical correlation is valid for $0 \leq \phi \leq 0.12$ , $10 \text{ nm} \leq d_p \leq 5 \mu\text{m}$ and $288 \text{ K} \leq T \leq 360 \text{ K}$ .
Arasu et al. [18]	$k_r = 0.9472 - 0.052\phi + 0.001482T + 0.00663(\phi T)$	The ranges of $T$ and $\phi$ are 308-333 K and 0.1-0.4% for TiO <sub>2</sub> with W.

Table 2. Various correlations on  $\mu_r$  nanofluids

Authors	Correlations	Remarks
Einstein [19]	$\mu_r = 1 + 2.5\phi$	The model is valid for low $\phi$ ( $\phi < 2\%$ ).
Graham [20]	$\mu_r = 1 + 2.5\phi + \frac{4.5}{\left(\frac{h}{a}\right)\left(2 + \frac{h}{a}\right)\left(1 + \frac{h}{a}\right)^2}$ $h/a = (2)[1 - (\phi/\phi_m)^{1/3}]/(\phi/\phi_m)^{1/3}$	$h$ is minimum distance separating two spheres. $a$ is radius of particle. $\phi_m$ is maximum packing spheres.
Chen et al. [21]	$\mu_r = \left(1 - \frac{\phi}{0.605} \left(\frac{a_a}{a}\right)^{1.2}\right)^{-1.5125}$	$d_p$ is 25 nm. The ranges of $T$ and $\phi$ are 293-333 K and 0-2% for TiO <sub>2</sub> -EG.
Masoumi et al. [22]	$\mu_r = 1 + \frac{\rho_p^{V_{br}} d_p^2}{72C\delta\mu_{bf}}$	$d_p$ is 13 and 28 nm. The ranges of $T$ and $\phi$ are 290-340 K and 1-5% for Al <sub>2</sub> O <sub>3</sub> -W.
Hosseini et al. [23]	$\mu_r = \exp\left[m + \alpha\left(\frac{T}{T_0}\right) + \beta(\phi_h) + \gamma\left(\frac{d_p}{1+r}\right)\right], \phi_h = \phi\left[\frac{d_p + 2s}{d_p}\right]^3$	$T_0 = 20$ °C. $d_p$ is 36 and 47 nm. The ranges of $T$ and $\phi$ are 293-333 K and 0-10% for Al <sub>2</sub> O <sub>3</sub> -W.
Corcione [12]	$\mu_r = \frac{1}{1 - 34.87\left(\frac{d_p}{d_{bf}}\right)^{-0.3} \phi^{1.03}}, d_{bf} = 0.1\left(\frac{6M}{N\pi\rho_{bf0}}\right)^{1/3}$	The ranges of $d_p$ , $T$ and $\phi$ are 25-200 nm, 293-333 K and 0.01-7% for Al <sub>2</sub> O <sub>3</sub> -W and propylene glycol, TiO <sub>2</sub> -W and EG, SiO <sub>2</sub> -ethanol and Cu-EG.
Adio et al. [24]	$\mu_r = 1 + \left[a_0 + a_1\left(\frac{T}{T_0}\right) + a_2\left(\frac{d_p}{h}\right)\right]\phi + \left[a_3\left(\frac{d_p}{h}\right)^2 + a_4\left(\frac{T}{T_0}\right)^2 + a_5\right]\phi^2 + a_6\left(\frac{T}{T_0}\right)^2 \phi^3$ $a_0 = 7.0764, a_1 = -0.1246, a_2 = -0.0346, a_3 = -0.0024, a_4 = -1.2357, a_5 = 53.6946, a_6 = 0.0436$	$d_p$ is 21, 105 and 125 nm. The range of $T$ is 293-343 K and $\phi \leq 5\%$ for MgO-EG.
Garossi [17]	$\mu_r = 1 + 49.6 \times \left(\frac{d_p}{d_{bf}}\right)^{-0.414} \phi^{0.908} e^{(10.8\phi)}$	Empirical correlation is valid for $0 \leq \phi \leq 0.12, 10 \text{ nm} \leq d_p \leq 5 \mu\text{m}$ .
Esfe et al. [27]	$\mu_r = 1.01171 + 0.0387\phi + 6.35 \times 10^{-5}T - 0.0010096\phi T - 0.00466\phi^2 + 2.127 \times 10^{-5}T^2 + 0.000107\phi^2 T + 4.527 \times 10^{-5}\phi T^2$	The ranges of $T$ and $\phi$ are 298-323 K and 1-4% for AlN (Aluminium nitride) with EG.

Table 3. Thermophysical properties of nanoparticles

Nanoparticles	$k$ (W/m K)	$\rho$ (kg/m <sup>3</sup> )
Al <sub>2</sub> O <sub>3</sub> [14]	40	3970
CuO [14]	20	6400
SiO <sub>2</sub> [14]	1.2	2200
TiO <sub>2</sub> [49]	8.95	4250
MgO [50]	48.4	3580
ZnO [51]	13	5600
Ag [17]	429	10500
Al [36]	237	2700
CNT, MWCNT [30, 52]	2000	2100

### 3. Results and Discussion

Figure 1 shows comparison of the predicted  $k_r$  values obtained from Equation (1) with experimental data including Al<sub>2</sub>O<sub>3</sub>-W [28, 29, 33], CuO-W [33], TiO<sub>2</sub>-W [32], Ag-W [35] and CNT-W [31] with a  $d_p$  range of 13-150 nm, a  $T$  range of 295-323 K, a  $\phi$  range of 0-18.1%.

Figure 2 illustrates comparison of the predicted  $k_r$  values obtained from Equation (1) with experimental data containing SiO<sub>2</sub>-EG [39], ZnO-EG [34], Al-EG [36], Al<sub>2</sub>O<sub>3</sub>-20:80%, 40-60%, 60:40% EG-W [38], MWCNT-30:70% EG-W [37] with a  $d_p$  range of 5-50 nm, a  $T$  range of 293-303 K, a  $\phi$  range of 0-5%.

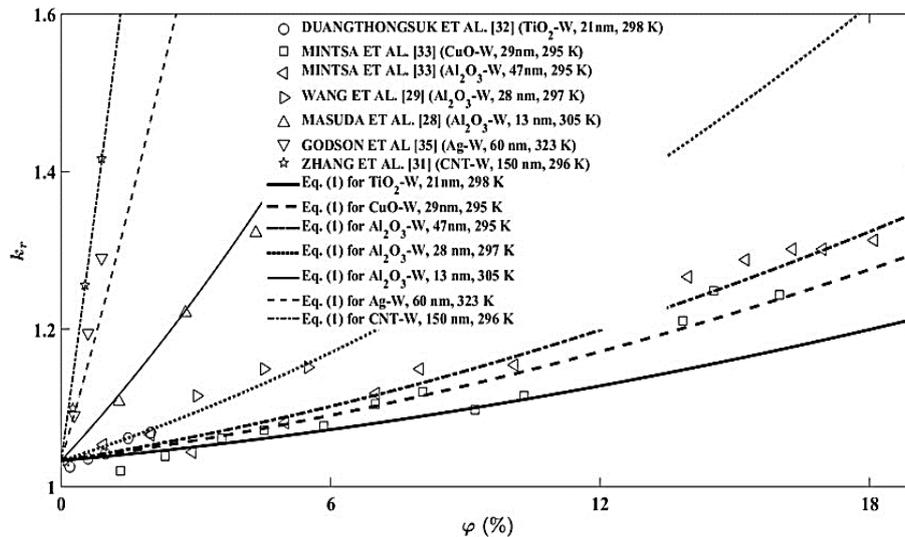


Figure 1. Comparison between the predicted values of  $k_r$  produced using Equation (1) and the values obtained by the experimental data of W based nanofluids

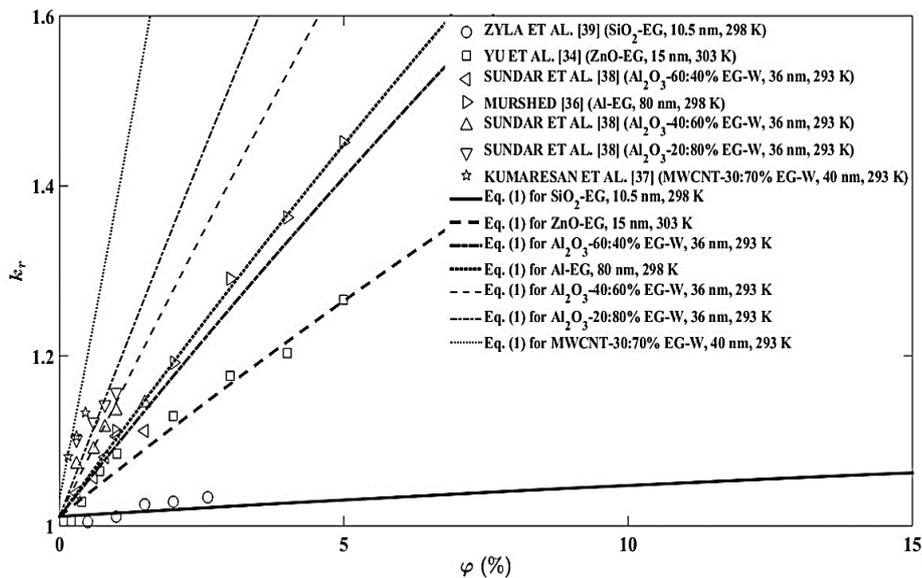


Figure 2. Comparison between the predicted values of  $k_r$  produced using Equation (1) and the values obtained by the experimental data of EG / EG-W based nanofluids

It is clear from Figures 1 and 2 that the predicted  $k_r$  values found using Equation (1) agree well with experimental data. Previous experimental studies illustrate that the type, shape, and size of nanoparticles affect  $k_r$  of nanofluids which generally increases with a reduction of nanoparticle's size. Figures 1 and 2 show that the suggested correlation estimates well  $k_r$  of nanofluids with different nanoparticle shapes, sizes, and types. The dimensionless parameters containing  $h$ ,  $d_p$ , and  $\psi$  in Equation (2) improve estimation of different size of spherical and cylindrical nanoparticles. The maximum values of  $k_r$  are gained by nanofluids with higher thermal conductive particles such as Ag, CNT and MWCNT as shown in Table 3.

Like the experimental data in Figures 1 and 2, the predicted  $k_r$  values obtained by Equation (1) increase when

decreasing  $d_p$  and increasing  $T$  and  $\phi$ . Moreover, dimensionless parameters including the nanoparticle's Brownian velocity and thermophysical properties of base fluids enhance predicting  $k_r$  of nanofluids with different weight ratios of EG-W mixtures as indicated in Figure 2.

In Figure 3, the predicted  $k_r$  values obtained using Equation (1) are compared with experimental  $k_r$  values of W, EG / EG-W based nanofluids. As shown in Figure 3, the estimated  $k_r$  values are very close to experimental  $k_r$  values with standard deviations of 2.16%.

In addition, 97.5% of the predicted data match experimental data with a mean deviation of  $\pm 5\%$  as demonstrated in Figure 3.

Figures 4 shows comparison of the estimated values obtained from Equation (6) with experimental data including  $\text{Al}_2\text{O}_3$ -W [28, 46], ZnO-W [44], Ag-W [35] and

MWCNT-W [45] with a  $d_p$  range of 9.2-75 nm, a  $T$  range of 293-323 K, a  $\phi$  range of 0-20.4%.

Figures 5 illustrates comparison of the estimated values obtained from Equation (6) with experimental data containing SiO<sub>2</sub>-EG [39], MgO-EG, [24] ZnO-EG [47], CuO -60:40% EG-W [43], MWCNT-50:50% EG-W [48] with a  $d_p$  range of 10.5-70 nm, a  $T$  range of 293-323 K, a  $\phi$  range of 0-6.2%, respectively. Figures 4 and 5 indicate that the calculated  $\mu_r$  values obtained Equation (6) are in good agreement with the experimental  $\mu_r$  values. As demonstrated in Figures 4 and 5, the estimated  $\mu_r$  values gained by Equation (6) confirm the general trends of  $\mu_r$  of nanofluids that the values of  $\mu_r$  increase when increasing  $\phi$  and decreasing  $d_p$  and  $T$ .

The present correlation also takes account of the impact of particle shape on  $\mu_r$  of nanofluids using  $\psi$  in Equation (7). Like the experimental data of  $\mu_r$  in Figures 4 and 5, the

maximum  $\mu_r$  values are obtained by nanofluids with cylindrical particles such as CNT and MWCNT. As demonstrated in Figure 5, the relative dynamic viscosity of nanofluids with different weight ratios of EG-W mixtures are predicted by using dimensionless parameters including  $u_{Br}$ ,  $\rho_{bf}$ , and  $\mu_{bf}$ .

In Figure 6, the predicted  $\mu_r$  values obtained from Equation (6) are compared with experimental  $\mu_r$  values of W, EG / EG-W based nanofluids. The predicted  $\mu_r$  values calculated using Equation (6) match closely with experimental  $\mu_r$  values of W, EG / EG-W based nanofluids with standard deviations of 8.16% as demonstrated in Figure 6. 90.4% and 96.2% of the predicted data suit experimental data with a mean deviation of  $\pm 10\%$  and  $\pm 20\%$ , respectively. Two points of the calculated data are above a 20% of deviation.

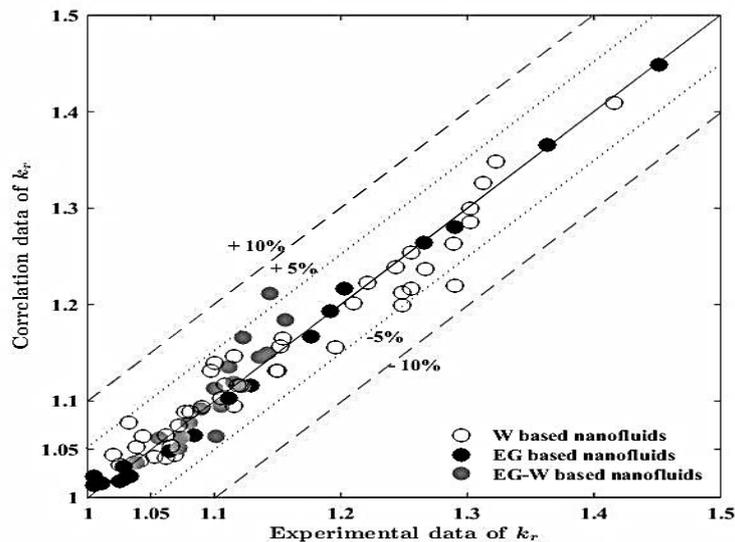


Figure 3. Comparison between the estimated values of  $k_r$  calculated using Equation (1) and the measured values of  $k_r$  for W, EG / EG-W based nanofluids

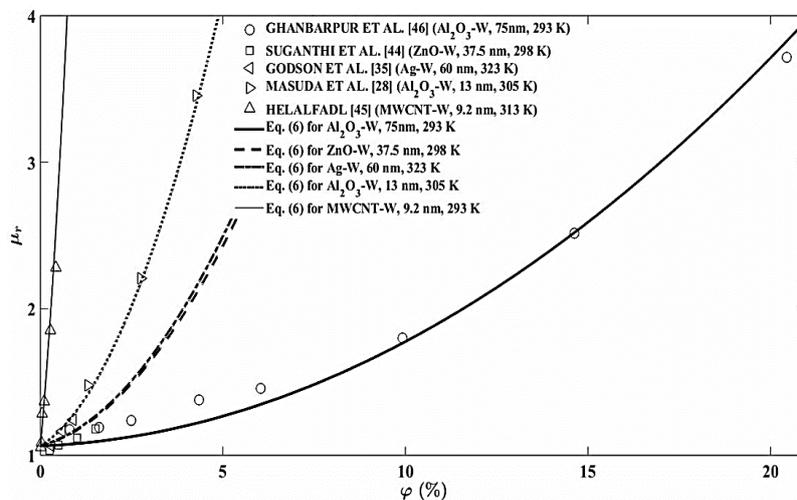


Figure 4. Comparison between the predicted values of  $\mu_r$  generated using Equation (6) and the values obtained by the experimental data of W based nanofluids

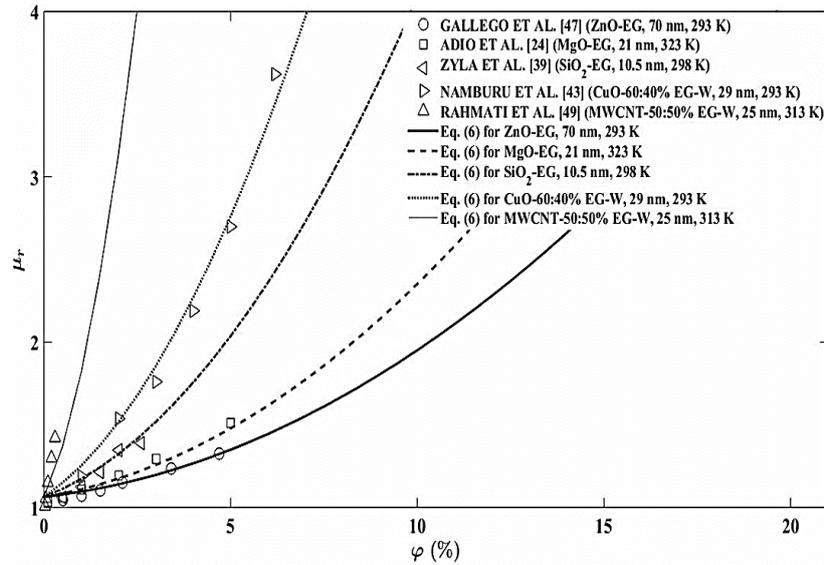


Figure 5. Comparison between the predicted values of  $\mu_r$  generated using Equation (6) and the values obtained by the experimental data of EG / EG-W based nanofluids

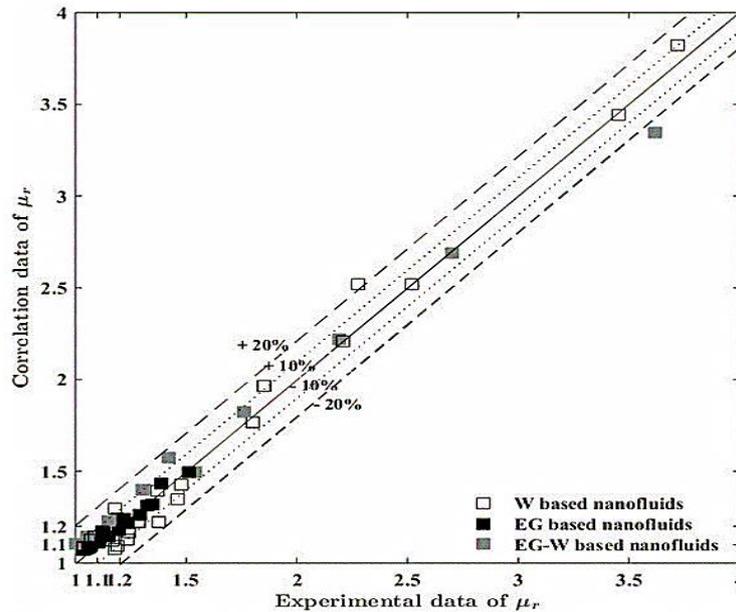


Figure 6. Comparison between the estimated values of  $\mu_r$  calculated using Equation (6) and the measured values of  $\mu_r$  for W, EG / EG-W based nanofluids

#### 4. Conclusions

In this paper,  $k_r$  and  $\mu_r$  of nanofluids are estimated by using the correlations with various dimensionless parameters related to the Brownian motion and properties of base fluids and nanoparticles as an extension study of [53]. The following results are achieved:

- Since  $k_r$  and  $\mu_r$  of nanofluids depend on many factors, increasing number of parameters instead of using only  $\phi$  and extending range of parameters in the correlations enhance accuracy of the results.
- Experimental results show that  $k_r$  generally increases when decreasing  $d_p$  and increasing  $k_p$ ,  $T$ , and  $\phi$  while  $\mu_r$  generally increases when increasing  $\phi$  and decreasing

$d_p$  and  $T$ . The suggested correlations suit this general trend.

- Adding  $1/\psi$  in Equations (2) and (7) helps to improve the estimation of  $k_r$  and  $\mu_r$  of nanofluids with cylindrical shape nanoparticles.
- The dimensionless parameters including particles Brownian motion and thermophysical properties of base fluids contribute to estimating  $k_r$  and  $\mu_r$  of nanofluids for different weight ratios of EG-W mixtures.
- Owing to improving the correctness of predicting  $k_r$  and  $\mu_r$  of nanofluids with spherical and cylindrical nanoparticles by using new dimensionless parameters,

Equations (1) and (6) will be useful tool in designing the nanofluids for various industrial applications.

### Declaration

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article. The authors also declared that this article is original, was prepared in accordance with international publication and research ethics, and ethical committee permission or any special permission is not required.

### Author Contributions

All authors conceived the study and developed the methodology. M. Kaplan performed the computations and wrote the manuscript. M. Özdiñç Çarpınlioğlu contributed to the final version of the manuscript.

### Nomenclature

$c_p$	: Specific heat, J/kg K
$d$	: Nanoparticle diameter, nm
$h$	: Capping layer thickness, Nm
$k$	: Thermal conductivity, W/m K
$k_b$	: Boltzmann constant, $1.3806 \times 10^{-23}$ J/K
$T$	: Temperature, K
$Pr$	: Prandtl number
$Re$	: Reynolds number
$u_{Br}$	: Brownian velocity (m/s)
$\beta$	: Empirical constant
$\Delta$	: Correlation term
$\eta$	: Empirical constant
$\mu$	: Dynamic viscosity, Pas
$\rho$	: Density ( $\text{g/cm}^3$ )
$\varphi$	: Volume fraction (%)
$\psi$	: Particle sphericity
$bf$	: Base fluid
$p$	: Particle
$r$	: Relative
$ref$	: Reference
$AlN$	: Aluminium nitride
$CNT$	: Carbon nanotubes
$EG$	: Ethylene glycol
$MWCNT$	: Multiwalled carbon nanotubes
$W$	: Water

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